

On the estimation of galaxy structural parameters: the Sérsic Model

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ABSTRACT

This paper addresses some questions which have arisen from the use of the Sérsic $r^{1/n}$ law in modelling the luminosity profiles of early type galaxies. The first issue deals with the trend between the half-light radius and the structural parameter n . We show that the correlation between these two parameters is not only real, but is a natural consequence from the previous relations found to exist between the model-independent parameters: total luminosity, effective radius and effective surface brightness. We also define a new galaxy concentration index which is largely independent of the image exposure depth, and monotonically related with n . The second question concerns the curious coincidence between the form of the Fundamental Plane and the coupling between $\langle I \rangle_e$ and r_e when modelling a light profile. We explain, through a mathematical analysis of the Sérsic law, why the quantity $r_e \langle I \rangle_e^{0.7}$ appears almost constant for an individual galaxy, regardless of the value of n (over a large range) adopted in the fit to the light profile. Consequently, Fundamental Planes of the form $r_e \langle I \rangle_e^{0.7} \propto \sigma_0^x$ (for any x , and where σ_0 is the central galaxy velocity dispersion) are insensitive to galaxy structure. Finally, we address the problematic issue of the use of model-dependent galaxy light profile parameters versus model-independent quantities for the half-light radii, mean surface brightness and total galaxy magnitude. The former implicitly assume that the light profile model can be extrapolated to infinity, while the latter quantities, in general, are derived from a signal-to-noise truncated profile. We quantify (mathematically) how these parameters change as one reduces the outer radius of an $r^{1/n}$ profile, and reveal how these can vary substantially when $n \geq 4$.

Key words: galaxies: elliptical and lenticular, cD – galaxies: fundamental parameters – galaxies: photometry – galaxies: structure – methods: data analysis – techniques: photometric.

1 INTRODUCTION

The parametrisation of galaxies is a staple activity of many astronomers. Indeed, it enables one to perform comparative studies and search for correlations which hopefully provide a deeper insight into the formative and evolutionary mechanisms at play in the Universe. Some of the most fundamental quantities pertaining to a galaxy come from the child-like questions: How big is it? How bright is it? One way astronomers answer such apparently simple questions is through fitting model profiles to the radial distribution of a galaxy's light. For many years the de Vaucouleurs (1948,

1959) $r^{1/4}$ law was employed for this task amongst the Elliptical galaxies. However, over the last decade or so, as the quality of the data has improved – largely due to the use of CCDs – this fitting function has been replaced by the generalised $r^{1/n}$ profile first proposed by Sérsic (1968), and revitalised by Capaccioli (1987, 1989), Davies et al. (1988) and Caon, Capaccioli & D'Onofrio (1993). In the case of Spiral galaxies, the central bulge is also well modelled with an $r^{1/n}$ profile (Andredakis, Peletier & Balcells 1995; Moriondo, Giovanardi & Hunt 1998; Khosroshahi, Wadadekar, & Kembhavi 2000; Graham 2001; Möllenhoff & Heidt 2001) – while an exponential disk model does a remarkably good job at matching the observed disk light distribution.

Although $r^{1/n}$ models fit the ‘observed’ light profiles very closely, there remains the question of where the galaxy

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actually finishes. In practice, the sky-background often limits the extent to which one has measured a galaxy, and the extrapolation, beyond which ever limiting isophote this may be, is somewhat problematic. Application of light profile models to surface brightness profiles implicitly assumes that the galaxy profiles extend to infinity. For a rapidly declining intensity profile this is not such a bad assumption, as the extra galaxy light beyond that actually observed is usually only a small percentage of the total galaxy light. However, in the case of the $r^{1/4}$ law, such extrapolation can be substantial. What this means is that the model derived parameters of size (the effective half-light radius r_e) and brightness (the surface brightness at this radius, μ_e , or the mean surface brightness enclosed by this radius, $\langle\mu>_e$), and total galaxy magnitude, can be significantly different to those obtained in a model-independent fashion using the ‘truncated’ galaxy light profile.

In Section 2, we commence by giving a little support to the $r^{1/n}$ models by highlighting an often over-looked fact. The correlation between the shape parameter n and galaxy size r_e , from a given sample of Elliptical galaxies, is definitely not explained by parameter coupling in the fitting process; this trend between galaxy structure and size exists when one uses model-independent values. We show, in Section 3, that the shape parameter n is monotonically related with the central galaxy light concentration. Another issue of importance is the bias in the galaxy parameters when one fits an $r^{1/4}$ law to a profile which is better described with a light profile having a shape parameter n which is different to 4. In Section 4, we explore this mathematically by constructing the equations that govern the ratio of parameters r_n/r_4 (effective radii from the respective models) and $I_n(0)/I_4(0)$ (central intensities) when one forces an $r^{1/4}$ model to an $r^{1/n}$ profile. We do this by deriving, and solving, the analytical expressions which govern the χ^2 value which one hopes to minimise when fitting a classical de Vaucouleurs model to an intensity profile with shape n . These ratios are computed here as a function of both n , and the radial range r/r_n to which one fits the $r^{1/4}$ model. As a result, we are able to explain why the product $<I>_e^\alpha r_e$, where $\alpha \sim 0.7$, appears constant, independent of whether or not one fitted an $r^{1/4}$ or an $r^{1/n}$ model – answering a frequently mentioned question about galaxy structure. In Section 5, we compute numerically the relative change to the galaxy parameters when one truncates the surface brightness profile at differing radii. We summarise our main conclusions in Section 6.

2 THE n - $\log r_e$ RELATION

This section addresses the question of whether or not parameter coupling in the $r^{1/n}$ model can account for, or has resulted in, artificial correlations between the photometric parameters (see for e.g. Kelson et al. 2000). One particular aspect of this potential problem is the correlation found between the Sérsic index n and the logarithm of the effective radius r_e for Elliptical galaxies (Caon et al. 1993). If this correlation is physical, it means that the light distribution in Elliptical galaxies varies with galaxy size: larger galaxies tend to be more centrally concentrated than smaller galaxies (n can be thought of as a central concentration parameter, see Section 3).

It is known that, irrespective of the true galaxy profile shape, in general the effective half-light radius derived from a fitted $r^{1/n}$ model will decrease and increase as the value of n does. Therefore, it is important to verify if the trend between galaxy size and light profile shape (that is, structure) is physical, or simply an illusion of the model fitting. We shall see shortly that such a relation between structure and size (r_e) is real (that is, is not dependent on any fitted light profile model), and was in fact already present, although somewhat hidden, in the correlations between the other global structural parameters.

The Sérsic $r^{1/n}$ radial intensity profile can be written as:

$$I(r) = I(0) \exp^{-b_n \left(\frac{r}{r_e}\right)^{\frac{1}{n}}}, \quad (1)$$

where $I(0)$ is the central intensity, and r_e is a scale radius. The quantity b_n is a function of the shape parameter n , and is chosen so that the scale radius encloses half of the total luminosity. A good approximation is $b_n = 2n - 0.324$ for $n \geq 1$; however, in this paper we have used the exact value derived from $\Gamma(2n) = 2\gamma(2n, b_n)$, where $\Gamma(a)$ and $\gamma(a, x)$ are the gamma function and the incomplete gamma function (Abramowitz & Stegun 1964).

Without fitting a light profile model, the most commonly measured galaxy parameters are: the total galaxy luminosity L_T , the effective radius r_e (defined as the equivalent radius of the isophote encircling half of the total galaxy flux), and the effective surface brightness μ_e . These can be measured from the direct integration of the flux out to some limiting isophote, and, if desired, subsequent extrapolation to infinity by some appropriate technique. These quantities are obtained without any assumption of a model (although some ad hoc hypothesis is often done for the extrapolation to infinity), and can be applied to any galaxy.

Observers have found, what are today, well known correlations between these photometric parameters. Figure 1 shows one example of these correlations – based on the data set of Elliptical galaxies in the Virgo and Fornax Clusters (data from Caon et al. 1990, 1994). The existence of such correlations implies (clearly) two things:

- the values are restricted to a finite region of the total parameter space, and
- the parameters are not independent of each other.

Now, for a general photometric model, we can write the total luminosity as:

$$L_T = k_L I_e r_e^2, \quad (2)$$

where k_L is a ‘structural parameter’ whose value depends on the form of the galaxy light distribution (Djorgovski, de Carvalho & Han 1988; Graham & Colless 1997). If we assume that the Sérsic model can provide a good description of Elliptical galaxies, we can identify the k_L term as

$$k_L = e^{b_n} \frac{2\pi n}{b_n^{2n}} \Gamma(2n). \quad (3)$$

Thus, to every triplet (L_T, I_e, r_e) of global parameters corresponds a unique value of k_L , and hence n . If all galaxies followed the $r^{1/4}$ law then n would equal 4 for every galaxy, and k_L would be constant for every galaxy.

In Figure 2, we have plotted the values of n and $\log r_e$. In the left panel, the value of r_e is that obtained from the model-independent analysis, and n comes from equation 3.

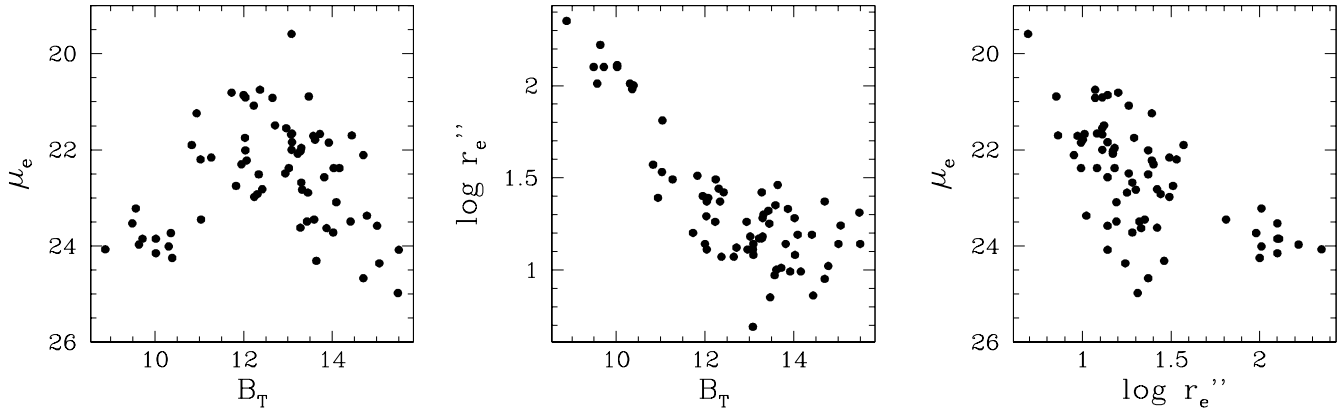


Figure 1. Correlations between the global structural parameters: total luminosity, effective radius and effective surface brightness. Data from Caon et al. (1990; 1994).

In the right panel, both r_e and n were obtained from fitting a Sérsic model to the surface brightness data. The agreement is clearly good. What this is telling us is that the structure of the Elliptical galaxies does depend on their size – independent of any fitted photometric model – one could have plotted k_L instead of n .

To re-iterate, independent of any model, the size of an Elliptical galaxy is related to its structure (as given by either the little used/known k_L term or the Sérsic shape parameter n).[†] If one chooses to characterise or model these structural shapes with the $r^{1/n}$ model, one finds a good agreement between the values obtained in a model-independent way. In other words, the relationship between n (or k_L) and $\log r_e$ is not simply due to parameter coupling in the fitting routine. Error coupling between n and r_e (Graham et al. 1996) can “stretch” the correlation, but can not account for it entirely.[‡]

When a model for the observed surface brightness profile is assumed, only galaxies which are well described by the model should be used to obtain quantitative information. This can limit the number of the galaxies analysed in a sample. So then, why is it necessary, and/or useful, to use a model profile to obtain photometric parameters? We can think of several reasons:

- (i) Although global, model-independent parameters can

be obtained for virtually any galaxy, their physical meaning can be uncertain for the different morphological classes, or rather, ill-defined for multiple component galaxies.

- (ii) A model provides a tool to disentangle different stellar components in galaxies (e.g. Sérsic bulge plus exponential disk).

- (iii) By using simple analytical functions to describe the light distribution, it is possible to analytically obtain the density profile using the Abel integral equation.

- (iv) It allows one to obtain a more reliable description of the over-all structure (value of n) by restricting the fit to the “good” part of the light profile, excluding, for instance, outer regions which can be affected by low S/N, sky-subtraction errors or distortions due to tidal disturbances.

3 A CENTRAL CONCENTRATION PARAMETER, AND ITS RELATION WITH n

Following Doi, Fukugita & Okamura (1993), the ‘concentration’ of a galaxy’s light is defined in Abraham et al. (1994) as

$$C = \frac{\sum_{i,j \in E(\alpha)} I_{ij}}{\sum_{i,j \in E(1)} I_{ij}}. \quad (4)$$

With the radius normalised to 1 at the outer measurable isophote, $E(\alpha)$ represents the isophote whose radius is α (<1) times that of the outer radius of the galaxy. I_{ij} represents the intensity in the pixel (i, j) .

This definition is clearly ill posed when dealing with a profile model which extends to infinity; $C=L(<\alpha r)/L(<r) \rightarrow 1$ as $r \rightarrow \infty$. But even with a profile that is truncated at some radius (and by this we include a radius that may encompasses all of the galaxy light), this definition still poses problems. The outer isophote to which one reliably detects light is a function of exposure time, telescope aperture, sky-brightness, personal signal-to-noise detection

[†] This correlation is still clear, albeit with larger scatter, if we plot n against the old-fashioned “effective aperture”: the radius of the circle centered on the nucleus within which one-half of the total flux is emitted – as listed in the RC3 catalog.

[‡] Total magnitudes and effective radii were measured by Caon et al. (1990, 1994) by using the $r^{1/4}$ model to extrapolate the growth curve to infinity. This means that L_T and r_e will be underestimated for profiles with $n > 4$, and over-estimated for those profiles with $n < 4$; this will shift the points in the $\log r_e$ – n diagram in the direction opposite to the parameter coupling in the Sérsic model.

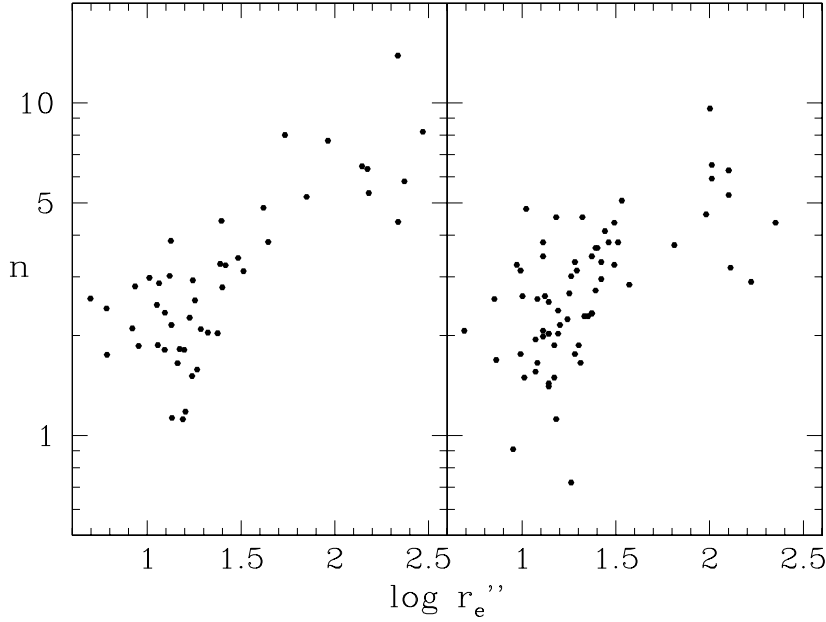


Figure 2. Left panel: correlation between n and r_e , where n is derived from the global parameter correlations (using equations 2 and 3); right panel: correlation between n and r_e using the values from the $r^{1/n}$ model fit to the light profile. (Data from Caon et al. 1993).

requirements, etc. Now while a 30% gain in radius, from a deeper image, may not necessarily change the total luminosity by much, it will change the normalised radius α by 30%, which can significantly effect the amount of light enclosed by this inner radius, and hence significantly effect the value of C .

We therefore propose a definition for the ‘central’ concentration which does not present this problem and can also be used in both a model-dependent and a model-independent way. Let C_{r_e} be the central concentration index, such that

$$C_{r_e}(\alpha) = \frac{\sum_{i,j \in E(\alpha r_e)} I_{ij}}{\sum_{i,j \in E(r_e)} I_{ij}}. \quad (5)$$

Here, $E(r_e)$ means the isophote which encloses half of the total light of the galaxy, and $E(\alpha r_e)$ is the isophote at a radius ($0 < \alpha < 1$) times r_e . This definition is still sensitive to the outer-most radius used to compute the total galaxy light, but not as strongly dependent as the definition of C given in equation 4. For a Sérsic law,

$$C_{r_e}(\alpha) = \frac{\gamma(2n, b_n \alpha^{1/n})}{\gamma(2n, b_n)}. \quad (6)$$

Figure 3 shows the values of C_{r_e} as a function of the Sérsic index n , for two different values of α , namely 0.3 and 0.5. The central concentration index and the index n are monotonically related, and n is therefore a useful estimator of the central concentration of a galaxy.

4 WHY $r_e \langle I \rangle_e^{0.7}$ IS CONSTANT – REGARDLESS OF n

When fitting a Sérsic model, the effective radius and mean intensity that one derives depends on the value of n . Different choices of n will give different values of r_e and $\langle I \rangle_e$ (and, of course, different χ^2 values for the fit); nevertheless, the product $r_e \langle I \rangle_e^\alpha$ with $\alpha \simeq 0.7$, is extremely stable[§] (Kormendy & Djorgovski 1989; Saglia, Bender, & Dressler 1993; Kelson et al. 2000). As a consequence, because the exponent α almost coincides with the exponent on $\langle I \rangle_e$ in the Fundamental Plane scaling law (Djorgovski & Davis 1987; Dressler et al. 1987) relating r_e and $\langle I \rangle_e$ with the central velocity dispersion σ , such that $\sigma^y \propto r_e \langle I \rangle_e^x$, the Fundamental Plane is quite independent of whether one adopts a Sérsic or a de Vaucouleurs law (Kelson et al. 2000)[¶]. Here, we show mathematically how the constancy of the above product, for an individual galaxy, is a direct consequence from the form of the Sérsic law.

To do this we construct a profile which is perfectly described by a Sérsic law with index n , effective radius r_n and central intensity $I_n(0)$. What happens if we then fit this profile with a de Vaucouleurs law; what values of r_4 and $I_4(0)$ shall we obtain?

[§] $\langle I \rangle_e$ means the average surface brightness within r_e . We will use the notation r_n and $\langle I \rangle_n$ for these quantities derived from a Sérsic profile with index n . $I_n(0)$ is the central intensity at $r=0$.

[¶] Kelson et al. 2000 varied the radial extent, and hence the data, to which they fitted their models each time they varied n , making a comparison of their structural parameters somewhat uncertain; the influence of the sky changing with each fitted model.

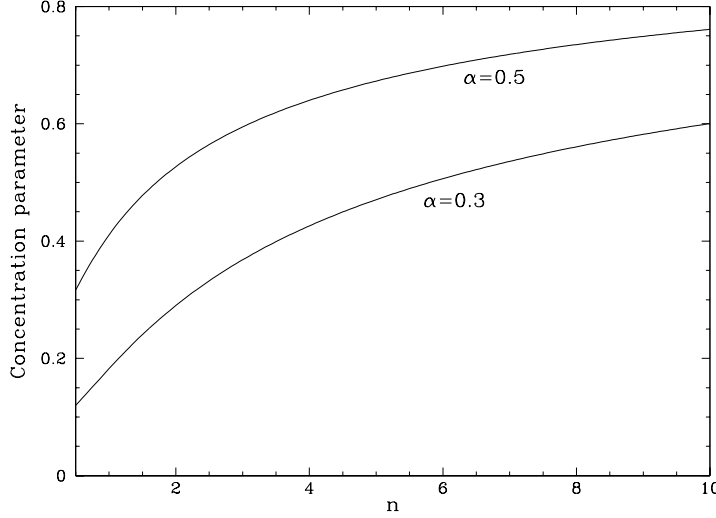


Figure 3. The central concentration parameter of galaxy light, as given in equation 5 and 6, is shown as a function of the Sérsic index n for different values of α .

Using the χ^2 goodness indicator for the fit, one can derive, after some work, two equations, one for each parameter, whose solutions provide the values of r_4 and $I_4(0)$ that minimise the χ^2 value, $\partial\chi^2/\partial I_4(0)=0$ and $\partial\chi^2/\partial r_4=0$. That is, we have derived the equations which minimise the χ^2 value for an $r^{1/4}$ profile from $r=0$ to $r=r_{\text{fin}}$ (some *final* outer radius).

These equations are, respectively,:

$$4\frac{r_4}{b_4^4}\gamma\left(4, b_4\left(\frac{r_{\text{fin}}}{r_n}x\right)^{1/4}\right) = \left(\frac{I_n(0)}{I_4(0)}\right)^2 S(r_{\text{fin}}, 1) \quad (7)$$

and

$$4\frac{r_4}{b_4^5}\gamma\left(5, b_4\left(\frac{r_{\text{fin}}}{r_n}x\right)^{1/4}\right) = \left(\frac{I_n(0)}{I_4(0)}\right)^2 S\left(r_{\text{fin}}, \left(\frac{r}{r_n}x\right)^{1/4}\right), \quad (8)$$

where

$$S(r_{\text{fin}}, f(r)) \equiv \int_0^{r_{\text{fin}}} e^{-2b_n\left(\frac{r}{r_n}\right)^{1/n} + b_4\left(\frac{r}{r_n}x\right)^{1/4}} f(r) dr, \quad (9)$$

$x \equiv r_n/r_4$ and r_{fin} denotes the outer radius of the fitted profile. From both equations we have derived, in equation 10, an implicit equation which gives the relation between r_n and r_4 as a function of n and r_{fin} . This equation is independent of the intensity.

$$S\left(r_{\text{fin}}, 1 - b_4 \frac{\gamma\left(4, b_4\left(\frac{r_{\text{fin}}}{r_n}x\right)^{1/4}\right)}{\gamma\left(5, b_4\left(\frac{r_{\text{fin}}}{r_n}x\right)^{1/4}\right)} \left(\frac{r}{r_n}x\right)^{1/4}\right) = 0 \quad (10)$$

In the limit, where $r_{\text{fin}} \rightarrow \infty$, this simplifies to:

$$S\left(\infty, 1 - b_4 \frac{\Gamma(4)}{\Gamma(5)} \left(\frac{r}{r_n}x\right)^{1/4}\right) = 0 \quad (11)$$

The solutions of this equation are shown in graphical form in Figure 4. The value of x can now be substituted into either equation 7 or 8 to obtain the ratio between the intensities. The result is shown in Figure 5.

We can now return our attention to the exponent α ;

from the previous results we are able to determine the value of α which solves the equation $\ln(r_n < I >_n^\alpha / r_4 < I >_4^\alpha) = 0$, and thereby keeps $r_n < I >_n^\alpha$ roughly constant. This is such that $\ln(r_n/r_4) + \alpha \ln(< I >_n / < I >_4) = 0$, which can be written as

$$\alpha(n) = - \frac{\ln x}{\ln \left[\frac{n}{4} \frac{b_4^8}{b_n^{2n}} \frac{\Gamma(2n)}{\Gamma(8)} \left(4 \frac{r_n}{x} \frac{1}{b_4^4} \frac{\gamma(4, b_4 \left(\frac{r_{\text{fin}}}{r_n}x\right)^{1/4}}{S(r_{\text{fin}}, 1)} \right)^{1/2} \right]}. \quad (12)$$

Figure 6 shows the value of α as a function of n for different r_{fin} . It is noted that the value of α appears to be more or less constant at ~ 0.7 , and only weakly dependent of n and r_{fin} . This explains why the Fundamental Plane relation, as mentioned previously, comes out pretty much the same irrespective of which model has been used. In passing, we note that this is not the only relation which appears stable; for example, $r_e^{1/\alpha} < I >_e$ is also stable.

While $r_e < I >_e^{0.7}$ is stable for individual galaxies, irrespective of the value of n used in the Sérsic model which derived these quantities, it seems to be a coincidence, rather than a natural consequence, that the same relation exists in the Fundamental Planes which have described the correlations between r_e , $< I >_e$ and central velocity dispersion for many different galaxies (Kelson et al. 2000). However, refined Fundamental Plane studies which have included the contribution from rotational energy, or used a global, rather than a central, velocity dispersion, have found an exponent of ~ 1 , in agreement with the expectation from the virial theorem (Busarello et al. 1997; Graham & Colless 1997; Prugniel & Simien 1997). We would therefore argue that for 'refined' Fundamental Plane studies, one should be concerned about the real range of structural shapes (as evidenced in Figure 2), and not be contempt with a model that ignores this; $r_e < I >_e^{1.0}$ is not invariable with n .

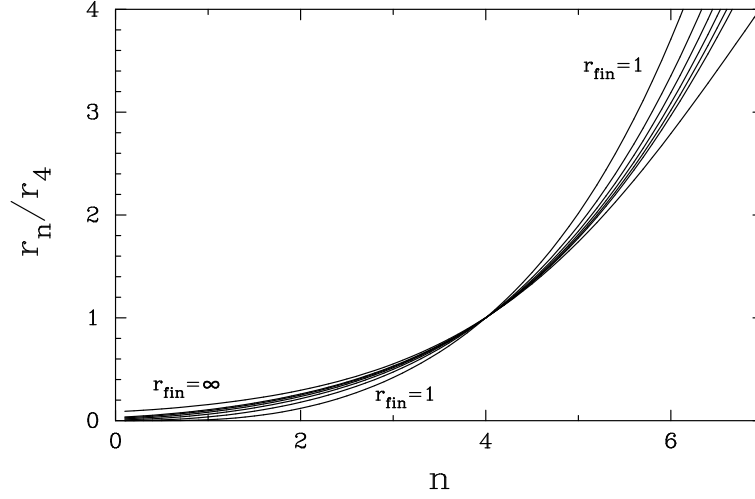


Figure 4. The ratio of effective radii r_n/r_4 when one forces an $r^{1/4}$ model to an $r^{1/n}$ profile for values of $r_{\text{fin}}=1, 2, 3, 4, 5, 6$ and ∞ .

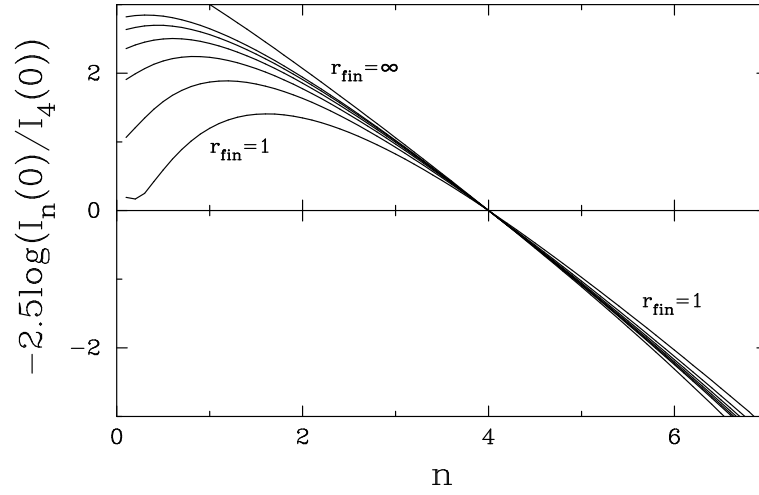


Figure 5. The ratio of central intensities $I_n(0)/I_4(0)$ when one forces an $r^{1/4}$ model to an $r^{1/n}$ profile for values of $r_{\text{fin}}=1, 2, 3, 4, 5, 6$ and ∞ .

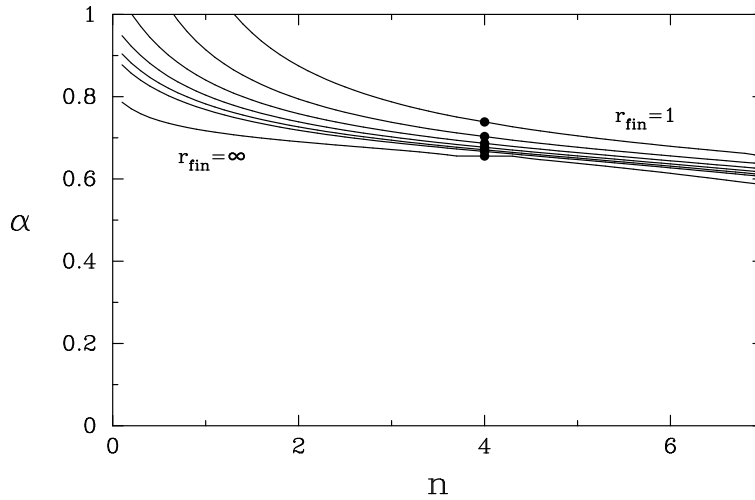


Figure 6. The α exponent from equation 12 (see text for explanation) for values $r_{\text{fin}}=1, 2, 3, 4, 5, 6$ and ∞ . α is indeterminate when $n=4$.

5 MODEL-DEPENDENT VERSUS MODEL-INDEPENDENT PARAMETRISATION

Although the photometric parameters L_T , r_e and $\langle\mu\rangle_e$ are, at least mathematically, well defined quantities, their values do depend on the method used to measure them. Not just with regard to the particular form of the fitting function (i.e. the model used to fit either the surface brightness profile, or the ‘curve of growth’), but also with regard to the radial extent to which the galaxy profile is assumed to hold.

The curve of growth can asymptote into the noise well before the galaxy actually peeters out – truncated by the sky-background and short exposure times. Summing the galaxy magnitude within the isophotal ellipse where this occurs can result in an under-estimation of the total galaxy magnitude, and hence an under-estimation to the effective half-light radius, and an over-estimate to intensity of the surface brightness term. On the other hand, assuming that the galaxy surface brightness profile follows, all the way to infinity, the best-fitting model to the inner data points, may result in the attribution of substantially more light to the galaxy than actually exists.

5.1 Total galaxy luminosity

The effective half-light radius derived from the Sérsic model will be denoted $r_{e,\text{mod}}$ from here on, to avoid confusion with the similar term derived in a model-independent way. The total luminosity associated with an $r^{1/n}$ model profile can be written as

$$L_T = I(0)r_{e,\text{mod}}^2 \frac{2\pi n}{b_n^{2n}} \Gamma(2n). \quad (13)$$

When using the integrated surface brightness profile, or growth curve, the observer selects a finite radius, r_{fin} , where the curve of growth becomes flat. The value of r_{fin} which one selects depends on the exposure time, and hence noise, in the outer parts of the galaxy image. If one accepts that the surface brightness profile out to r_{fin} is well described by a Sérsic law, contributing zero light at larger radii, then the ‘total’ luminosity obtained from direct measurements of the integrated surface brightness profile is

$$L(r_{\text{fin}}) = I(0)r_{e,\text{mod}}^2 \frac{2\pi n}{b_n^{2n}} \gamma \left(2n, b_n \left(\frac{r_{\text{fin}}}{r_{e,\text{mod}}} \right)^{\frac{1}{n}} \right). \quad (14)$$

For an $r^{1/n}$ profile that extends to infinity, the outer fraction of the total galaxy light beyond the radius r_{fin} is given by

$$F(r_{\text{fin}}) \equiv \frac{L_T - L(r_{\text{fin}})}{L_T} = 1 - \frac{\gamma(2n, b_n(\frac{r_{\text{fin}}}{r_{e,\text{mod}}})^{\frac{1}{n}})}{\Gamma(2n)} \quad (15)$$

This fractional difference to the total luminosity is plotted in Figure 7, as a function of $r_{\text{fin}}/r_{e,\text{mod}}$, for different values of n . When $r_{\text{fin}}/r_{e,\text{mod}}=1$, the fraction of the total galaxy luminosity outside r_{fin} is, by definition, 50 per cent (or 0.75 mag). For the de Vaucouleurs profile ($n=4$) the outer fraction to the luminosity is ~ 15 per cent (0.18 mag) at a radius r_{fin} equal to four $r_{e,\text{mod}}$.

Usually an observer knows, either in advance when preparing the observations, or after extraction of the light

profiles, the surface brightness limit μ_L of the image (that is, the surface brightness level of the last measurable isophote). For this reason, it is useful to plot (Figure 8) the parameter $F(r_{\text{fin}})$ against the difference $\mu_L - \mu_e$ for various values of n . Here, $F(r_{\text{fin}})$ is such that

$$F(r_{\text{fin}}) = 1 - \frac{\gamma(2n, b_n + \frac{\ln(10)}{2.5}(\mu_L - \mu_e))}{\Gamma(2n)} \quad (16)$$

It is evident that, to reach the same value of $F(r_{\text{fin}})$, the difference $\mu_L - \mu_e$ must be larger (that is, the images deeper) for bigger n . This is particularly important because μ_e and n are positively (though weakly) correlated. For instance, the μ_e - n plot from Caon et al. (1993) shows that, when $n = 2$, $\mu_e \simeq 22.5$, while at $n=8$ $\mu_e \simeq 23.5$ (in B). The growth curve integrated out to a limiting surface brightness of $\mu_L = 27$ mag/arcsec² will miss about 5% of the total luminosity when $n=2$, and 22% when $n=8$.

5.2 Effective radius

The two different estimates to the total galaxy luminosity (Eq. 13 and Eq. 14) result in two different estimates for the effective half-light radius. Taking the truncated profile, we define the observed effective half light radius $r_{e,\text{obs}}$ such that

$$L(r_{e,\text{obs}}) \equiv \frac{L(r_{\text{fin}})}{2}. \quad (17)$$

This can be expanded to give

$$2\gamma(2n, b_n(\frac{r_{e,\text{obs}}}{r_{e,\text{mod}}})^{\frac{1}{n}}) = \gamma(2n, b_n(\frac{r_{\text{fin}}}{r_{e,\text{mod}}})^{\frac{1}{n}}). \quad (18)$$

Once the value of r_{fin} is selected, and $r_{e,\text{mod}}$ and n derived, one can compute the ratio between $r_{e,\text{obs}}$ and $r_{e,\text{mod}}$. Figure 9 shows this ratio for different values of n and $r_{\text{fin}}/r_{e,\text{mod}}$. As n increases, for a fixed $r_{\text{fin}}/r_{e,\text{mod}}$ ratio, the $r_{e,\text{obs}}/r_{e,\text{mod}}$ ratio decreases from 1. For a de Vaucouleurs profile observed out to $4r_{e,\text{mod}}$, the radius containing half of the ‘observed’ galaxy light, $r_{e,\text{obs}}$, is 3/4 of the radius, $r_{e,\text{mod}}$, coming from the model extrapolation to infinity.

5.3 Effective surface brightness

The Sérsic surface brightness profile $\mu(r)$ can be written as

$$\mu(r) = \mu_0 + \frac{2.5b_n}{\ln(10)} \left(\frac{r}{r_{e,\text{mod}}} \right)^{1/n}, \quad (19)$$

where μ_0 is the central surface brightness. The effective surface brightness, μ_e , is the surface brightness at the effective half-light radius. Therefore, for an $r^{1/n}$ model which extends to infinity, it is given by $\mu_{e,\text{mod}} = \mu_0 + 2.5b_n/\ln(10)$. The difference between the model value and the value assuming a truncated profile is

$$\Delta\mu_e = \frac{2.5b_n}{\ln(10)} \left[1 - \left(\frac{r_{e,\text{obs}}}{r_{e,\text{mod}}} \right)^{1/n} \right]. \quad (20)$$

The mean effective surface brightness is defined as:

$$\langle\mu\rangle_e \equiv -2.5 \log \frac{L(r_e)}{\pi r_e^2}, \quad (21)$$

where r_e can be either $r_{e,\text{mod}}$ or $r_{e,\text{obs}}$. The difference, $\Delta\langle\mu\rangle_e \equiv \langle\mu\rangle_{e,\text{mod}} - \langle\mu\rangle_{e,\text{obs}}$ is given by the expression

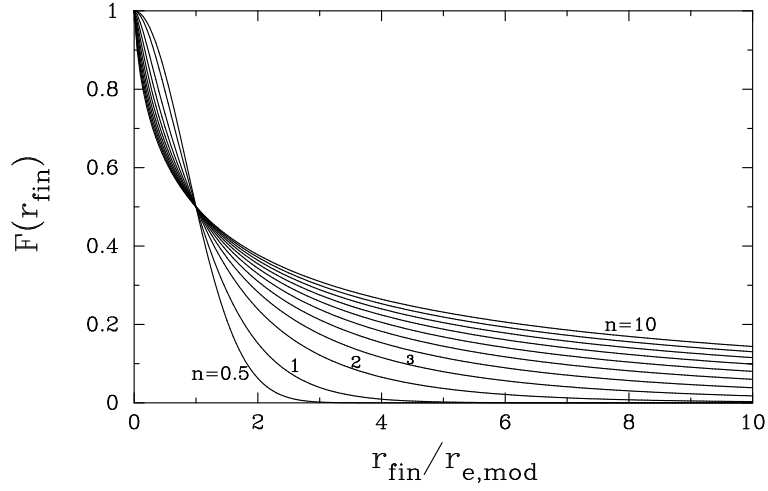


Figure 7. The parameter $F(r_{\text{fin}})$ is shown as a function of $r_{\text{fin}}/r_{e,\text{mod}}$ for values of $n=0.5, 1, 2, 3, \dots, 10$.

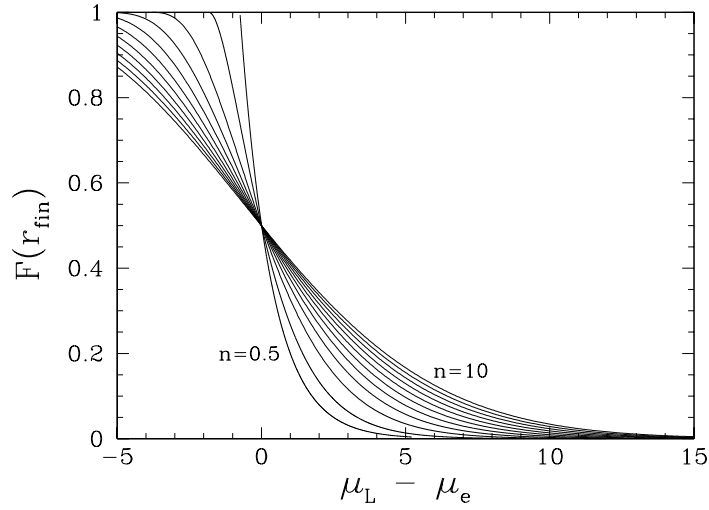


Figure 8. The parameter $F(r_{\text{fin}})$ is shown as a function of $\mu_L - \mu_e$ for values of $n = 0.5, 1, 2, 3, \dots, 10$.

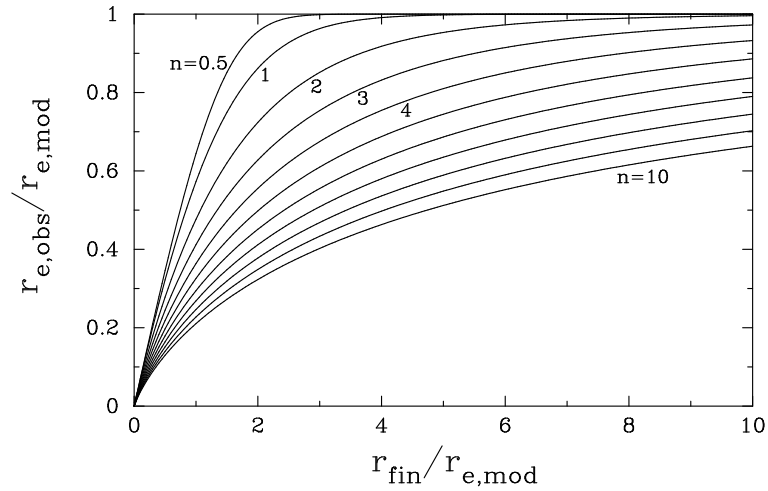


Figure 9. The $r_{e,\text{obs}}/r_{e,\text{mod}}$ ratio as a function of $r_{\text{fin}}/r_{e,\text{mod}}$ for different values of n (see the text for an explanation of terms).

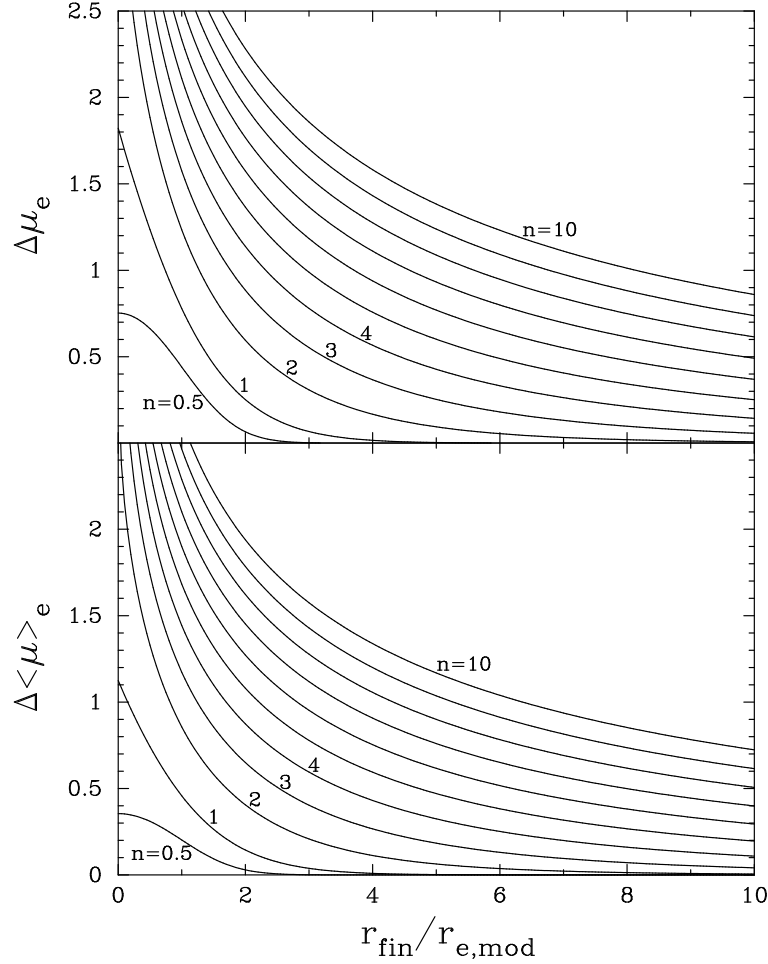


Figure 10. The difference in the effective surface brightness μ_e (upper panel) and mean effective surface brightness $\langle\mu\rangle_e$ as a function of $r_{\text{fin}}/r_{e,\text{mod}}$ for different values of n .

$$\Delta\langle\mu\rangle_e = -2.5 \log \left[\frac{\Gamma(2n)}{2\gamma(2n, b_n(\frac{r_{e,\text{obs}}}{r_{e,\text{mod}}})^{\frac{1}{n}})} \left(\frac{r_{e,\text{obs}}}{r_{e,\text{mod}}} \right)^2 \right] \quad (22)$$

Figure 10 shows both $\Delta\mu_e$ and $\Delta\langle\mu\rangle_e$ as a function of $r_{\text{fin}}/r_{e,\text{mod}}$ for different values of n .

6 CONCLUSIONS

We have shown that the correlation between Elliptical galaxy size (r_e) and luminous structure (n , or k_L) is not only real, but was already contained in the known correlations between the other global, model-independent photometric parameters: L_T , r_e and I_e . While parameter coupling in the fitting process can certainly contribute to the observed correlation, it can not account for it.

We have redefined the galaxy concentration index, rendering it almost independent of the limiting magnitude and/or radius of the galaxian map. For a Sérsic model, the new index displays a monotonic behaviour with n .

We have shown, through a mathematical analysis, why the quantity $\langle I \rangle_e^\alpha r_e$, with $\alpha \sim 0.7$, is fairly constant and insensitive to both n and the radius out to which the profile is fitted. Intriguingly, because α is practically equal to the exponent on the intensity term in the Fundamental Planes

constructed with the quantities r_e , $\langle I \rangle_e$ and central velocity dispersion, these Fundamental Planes are insensitive to galaxy structure. However, more refined studies which further take into account dynamical non-homology, rotational energy and/or consider metallicity effects, find a different exponent. Consequently, the use of an $r^{1/4}$ law to obtain r_e and $\langle I \rangle$ can affect these Fundamental Plane relations.

Galaxian images are generally limited in their radial extension by noise, sky subtraction errors or distortions in the outer parts; it is therefore useful to know how the photometric parameters vary as a function of the radius out to which the surface brightness distribution is integrated. Galaxies described with larger shape parameters (n) require more extended (in units of r_e), or deeper (with respect to μ_e), observations in order not to miss an important fraction of the total light. For example, when $n \geq 4$ one will obtain significantly smaller effective radii (by a factor of 2 or more) and brighter effective surface brightnesses (by 1 mag or more) if one only integrates to ~ 2 effective radii.

Unfortunately, the radial extent to galaxian images cannot simply be increased as far as one desires by integrating for longer, even if galaxies are of infinite radius. If the limiting factor were the photon noise from a homogeneous sky background, then the signal-to-noise ratio would be propor-

tional to the square root of the integration time. However, a further contribution to the sky background noise is given by scattered light and by faint sources which make up the extragalactic background light (a vivid representation is provided by imagining a galaxy superposed on the Hubble Deep Field). Dalcanton and Bernstein (2000) carried out a full analysis of the sky background noise sources and of the limiting surface brightness which can be achieved by deep imaging, finding limits of $\mu_B \sim 29.5$ mag arcsec⁻² and $\mu_R \sim 29$ mag arcsec⁻². This corresponds to ~ 10 effective radii for a bright elliptical with $\mu_e(B) \simeq 23.5$ and $n = 8$, and ~ 7 effective radii for a low-luminosity elliptical with $\mu_e(B) \simeq 22.5$ and $n = 2$.

In Caon et al. (1990, 1994) the light profiles of elliptical galaxies, reach a surface brightness limit of $\mu_B \simeq 27.5$ (typically 4–5 effective radii). There is no evidence for an outer truncation similar to that observed in the disks of spirals (Pohlen, Dettmar & Lütticke 2000 and references therein). As far as we know, nobody so far has detected definite edges in elliptical galaxies.

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